

Fractal network dimension and viscoelastic powerlaw behavior: I. A modeling approach based on a coarse-graining procedure combined with shear oscillatory rheometry

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Abstract

Recent advances in dynamic elastography and biorheology have revealed that the complex shear modulus, G^* , of various biological soft tissues obeys a frequency-dependent powerlaw. This viscoelastic powerlaw behavior implies that mechanical properties are communicated in tissue across the continuum of scales from microscopic to macroscopic. For deriving constitutive constants from the dispersion of G^* in a biological tissue, a hierarchical fractal model is introduced that accounts for multiscale networks. Effective-media powerlaw constants are derived by a constitutive law based on cross-linked viscoelastic clusters embedded in a rigid environment. The spatial variation of G^* is considered at each level of hierarchy by an iterative coarse-graining procedure. The establishment of cross-links in this model network is associated with an increasing fractal dimension and an increasing viscoelastic powerlaw exponent. This fundamental relationship between shear modulus dynamics and fractal dimension of the mechanical network in tissue is experimentally reproduced in phantoms by applying shear oscillatory rheometry to layers of tangled paper strips embedded in agarose gel. Both model and experiments demonstrate the sensitivity of G^* to the density of the mechanical network in tissue, corroborating disease-related alterations of the viscoelastic powerlaw exponent in human parenchyma demonstrated by *in vivo* elastography.

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